Yuhao Li (Dated: October 4, 2024)

- 1. Eq. (13) in page 2: The first item on the exponential should be  $-\frac{1}{2}\beta J_0 M m_{\alpha}^2$ .
- 2. The line below Eq. (25) in page 4: It might be Now as  $n \to 0, \cdots$
- 3. Eq. (37) in page 5: The factor  $\frac{1}{n}$  is missing in the right-hand side of the equal sign.
- 4. Eq. (47) in page 6: The sign of integration in the third term is missing. Besides, in Eq. (47), it seems that the equivalence relation

$$\frac{\mathrm{d}}{\mathrm{d}x}\tanh(\beta x) \xrightarrow{\beta \to \infty} 2\delta(x)$$

is used to obtain the third term from the second term. However, I don't know if this limit is correct, or if it's done in some other way.

5. Eq. (51) in page 6:

I believe that the result of the last integral should not use the approximately equal sign, omitting the coefficient  $1/\sqrt{2\pi}$ , since it is important for the value of the ground state entropy (although this does not change the sign).

- 6. The equation at the bottom of page 10: The second term  $-\frac{n}{2}\beta^2 J^2 q_0$  on the exponential in the second line and the first item  $-\frac{1}{2}\beta^2 J^2 q_0$  in the third line should be  $q_1$  instead of  $q_0$ .
- 7. Eq. (74) ~ (86) in page 11: The item  $\beta J \sqrt{q_0} u$  on the exponential of all these equations should be corrected to  $J \sqrt{q_0} u$  without an additional  $\beta$ .
- 8. Eq. (75) in page 11: The variable  $S^{\alpha}$  is missing on the exponential.
- 9. Eq. (96) in page 12: The sign before the last term  $\sum_i \lambda_i m_i$  should be + rather than -.

10. The equation at the bottom of page 12: The third and fifth lines are repeated with the second and fourth lines respectively. I guess you want to express

$$=\sum_{i}\lambda_{i}m_{i}-T\log\prod_{i}\sum_{s}\exp\left(\beta\lambda_{i}S_{i}\right)$$

on the third line.

11. Eq. (102) and (103) in page 13:

A square bracket should be placed after the second summation sign to indicate the summation of all subsequent terms, to avoid misunderstandings.